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Optimal power flow tool for hybrid AC/DC systems

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Abstract

This study presents a tool for solving the Optimal Power Flow (OPF) problem in mixed Direct Current (DC) and Alternating Current (AC) systems. It allows to analyse the optimal operation of multiple independent systems, DC connected and linked to the AC system, from a steady state point of view and for several objective functions.
The tool, implemented through implemented through MATLAB® Optimization toolbox is benchmarked with another OPF tool, implemented in MATPOWER®. Both tools lead to the same results for the system analysed when minimising losses and one minimising the deviation from a preset voltage profile. A sensitivity analysis is performed to assess the influence of DC and AC grid parameters on the operation of the system.

1 Introduction

In order to achieve the envisioned extensive renewable power integration, a future transmission network consisting of both High Voltage Direct Current (HVDC) and High Voltage Alternating Current (HVAC) grids is considered both probable and feasible [1]. HVDC technology is seen as specifically suitable to deliver the power produced in the wind power plants to the grid. When considering remote wind power plants to be connected to different AC grids, the HVDC system could be either a multi-terminal or a meshed DC grid [2–7].
Many technical and economic challenges exist around the design, planning, operation and control of HVDC-HVAC grids [3]. Regarding the design, two main converter technologies are available: VSC and LCC (Line Commutated Converter), where the first option is the most attractive [8]. New converter topologies, like the MMC (Modular Multilevel Converters) are being proposed, offering some advantages compared to previous VSC technologies. Mainly, lower frequency modulation, lower total harmonic distortion, lower losses and a modular structure, which allows scalability to different power and voltage levels [9]. In this sense, the voltage level and power rating definition of future HVDC interconnectors are also an issue. The need of meshing the DC grid is also being investigated, together with its protection and grounding [10]. Planning an appropriate expansion of the transmission system by merging HVDC links with the existing HVAC system requires a technical and economic analysis [11]. Some regulatory issues in terms of governance, ownership and grid codes still need to be defined. Even if all the aforementioned challenges are addressed, the operation of hybrid HVDC-HVAC grids still needs to be determined because it changes the way power systems are working and being controlled. This study proposes a tool for the optimal operation of these systems.
Although there are many studies on the operation of AC grids and DC grids separately, only a few have been published analysing the operation of both grids combined. In [12], an algorithm for solving power flows in AC/DC networks is presented and implemented in MATPOWER®, taking into account converter losses through a generalized converter loss model proposed in [13].
Some papers analyse the optimal operation of hybrid AC/DC systems, [14–18]. In [14], some power system elements are represented with limited accuracy: terminal VSC losses are neglected and only point-to-point connections are defined. Converter losses are included in [15–18]. [15] focuses on the mathematical formulation of the problem. The problem is reformulated for AC grids with embedded DC networks based on Second Order Cone Programming, which converts it into a convex problem. However, the optimal operation is only addressed in terms of loss minimization. Similarly, the authors of [16] only deal with transmission losses. But it is worth mentioning that [16] includes additional constraints reflecting grid code requirements, neglected in other studies. The authors' analysis from [17, 18] cover different optimization goals. Among all, the tool presented in [18] is the first one that defines and optimizes, for different objective functions, DC and AC load flows simultaneously in a random AC-DC network, allowing the possibility of meshing the DC system. It is implemented in MATPOWER®.
The tool in the present study has similar features to [18], but is implemented through MATLAB® Optimization toolbox (function *fmincon*).
On the other hand, this tool was already applied to a probable future scenario: a system with large integration of offshore wind, in [19], where it was proven that the results obtained when minimizing losses in a 4 DC 5 AC node system with two different softwares (GAMS® and MATLAB®) coincided.
This paper is organised as follows. The mathematical formulation of the tool is first presented in Section 2. Then, the tool is applied to a 3 DC, 5 AC bus network and benchmarked with the

alternative implementation from [18], for two different objective functions. The first one is loss minimization. The second objective function tested is minimum deviation from a preset voltage profile. In both study cases, the results coincide with the output from [18]. Therefore, the tool has been validated, offering a flexible methodology for analysing hybrid AC/DC grids optimal operation. Finally, a sensitivity analysis is included to show the effect of grid parameters on the optimal operation of the system.

2 Optimization problem

The optimization problem analysed involves DC and AC power systems and can, therefore, be classified as a non-linear constrained optimization. The solver chosen is the Interior Point Algorithm using a barrier function [20]. The layout of a general HVDC-HVAC system to which the tool is applied is shown in Figure 1. The converter topology is Voltage Source Converter (VSC), allowing an independent control of active and reactive power. VSCs operating as rectifiers inject the power generated by renewable sources (for instance, offshore wind power plants) into the DC grid. This power is transmitted to the AC grid through VSCs operating as inverters. Then, the power flows through the HVAC links until it reaches the consumption nodes.

The active power on the AC side and DC side of the converter differ on losses. So, the AC power can be defined as a function of the DC power and converter losses, modelled according to a second order polynomial of the AC converter current. DC cables are modelled through their resistances, AC cables are represented according to their π equivalent and transformers are modelled as an equivalent impedance with inductive and resistive part.

The active and reactive power demands in the AC nodes and the injections from the renewable sources are an input for the tool. The electrical characteristics of the DC and AC grids, as well as the distances of lines and cables and the converter loss parameters are also known data. The tool determines the active and reactive power injections (or absorptions) from generators and converters and the power flowing through each branch that minimize a specified objective function while accomplishing the electrical system constraints.

2.1 Notation

All the variables and parameters required for the mathematical formulation of the problem are listed below.

- G_{DC} conductance matrix of the DC grid
- G_{AC} conductance matrix of the AC grid
- B_{AC} susceptance matrix of the AC grid
- $i \in (1, n)$, n is the number of VSC converters
- $j \in (1, p)$, p is the number of AC nodes
- $\mathbf{I} = [I_1 \cdots I_n]^T$ vector of DC currents
- $\mathbf{E} = [E_1 \cdots E_n]^T$ vector of DC voltages
- $\mathbf{V} = [V_1 \cdots V_p]^T$ vector of AC voltage magnitude
- $\delta = [\delta_1 \cdots \delta_p]^T$ vector of AC voltage angles
- E_i and I_i are, respectively, the DC voltage and the current in node i.
- V_j and δ_j are, respectively, the AC voltage magnitude and angle of voltage phasor in node j.
- $\mathbf{P_{DC}} = [P_{DC_1} \cdots P_{DC_n}]^T$ is the power entering the DC system through the converters
- $\mathbf{P_g} = \left[P_{g_1} \cdots P_{g_p}\right]^T$ is the active power generated in each AC node
- $\mathbf{P_d} = \left[P_{d_1} \cdots P_{d_p}\right]^T$ is the active power demanded in each AC node
- $\mathbf{Q_{vsc}} = [Q_1 \cdots Q_n]^T$ is the reactive power injection/absorption by each converter
- $\mathbf{Q_g} = \left[Q_{g_1} \cdots Q_{g_p}\right]^T$ is the reactive power generated in each AC node
- $\mathbf{Q_d} = \left[Q_{d_1} \cdots Q_{d_p}\right]^T$ is the reactive power demanded in each AC node
- $\mathbf{S_{vsc}} = [S_1 \cdots S_n]^T$ is the power rating of each converter

2.2 Inputs

The input data for the optimization problem is the listed below:

- Conductance matrix of the DC grid: G_{DC}
- Conductance and susceptance matrix of the AC grid: G_{AC} and B_{AC}
- Active and reactive power demand in the AC grid nodes: P_d and Q_d
- Converter loss parameters

2.3 Outputs

The optimization algorithm determines the voltages in all the nodes and the power flowing in the different branches of the system that minimize a user defined objective function and guarantee all the equality and inequality constraints. Therefore, the output vector of the algorithm, x, contains the following information:

$$x = \begin{bmatrix} E \\ I \\ V \\ \delta \\ P \\ Q \end{bmatrix} \tag{1}$$

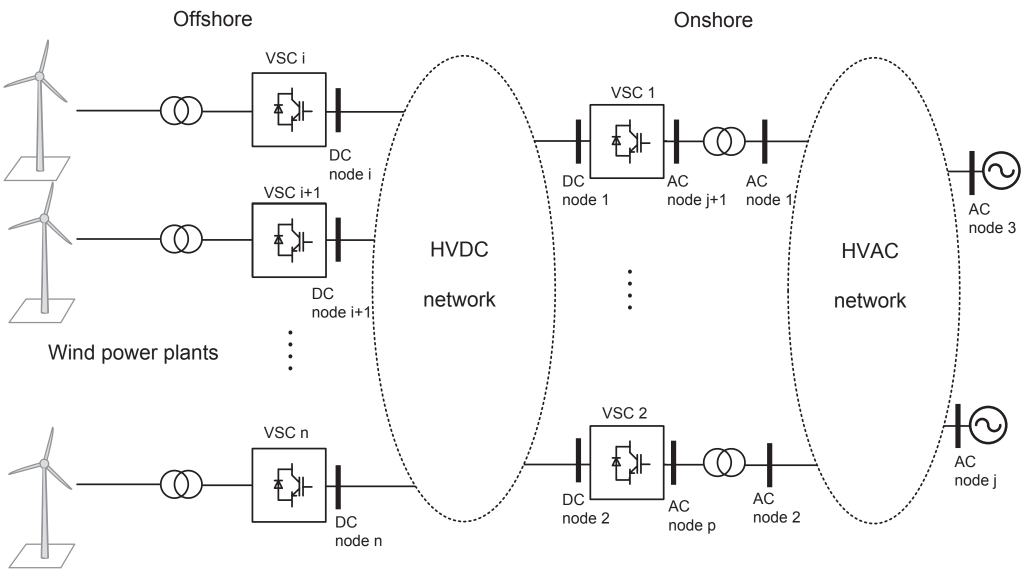

Fig. 1: Hybrid HVDC-HVAC system for integrating offshore wind power

2.4 Mathematical formulation

The problem is here formulated taking a general objective function, $f(x)$, which is dependent on the variables defined in (1).

$$[MIN]z = f(x) \tag{2}$$

subject to the following constraints:

$$\mathbf{I} = \mathbf{G_{DC}E} \tag{3}$$

$$P_{DC_i} = E_i I_i \tag{4}$$

$$P_{DC_i} - P_{lossvsc_i} = P_j + P_{g_j} - P_{d_j} \tag{5}$$

$$Q_j = Q_{vsc_j} + Q_{g_j} - Q_{d_j} \tag{6}$$

$$E_i^{min} \leq E_i \leq E_i^{max} \tag{7}$$

$$I_i^{min} \leq I_i \leq I_i^{max} \tag{8}$$

$$P_{kl}^{min} \leq G_{kl}\left(E_k - E_l\right) E_k \leq P_{kl}^{max} \tag{9}$$

$$V_j^{min} \leq V_j \leq V_j^{max} \tag{10}$$

$$P_j^{min} \leq P_j \leq P_j^{max} \tag{11}$$

$$Q_j^{min} \leq Q_j \leq Q_j^{max} \tag{12}$$

$$S_{kl}^{min} \leq S_{kl} \leq S_{kl}^{max} \tag{13}$$

$$\delta_j^{min} \leq \delta_j \leq \delta_j^{max} \tag{14}$$

$$S_{vsc_i}^{min} \leq S_{vsc} \leq S_{vsc_i}^{max} \tag{15}$$

being

$$P_j = V_j \sum_{k=1}^{p} V_k \left(G_{ACjk} \cos \delta_{jk} + B_{ACjk} \sin \delta_{jk}\right) \tag{16}$$

$$Q_j = V_j \sum_{k=1}^{p} V_k \left(G_{ACjk} \sin \delta_{jk} - B_{ACjk} \cos \delta_{jk}\right) \tag{17}$$

The AC links are modelled according to the π equivalent diagram, as sketched in Figure 2.

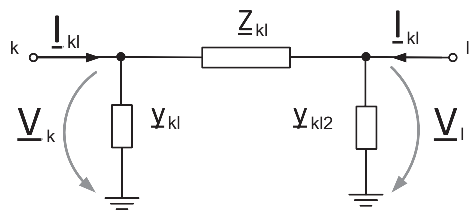

Fig. 2: π equivalent of the AC branch between nodes k and l

2.5 Converter model

The converter topology chosen for this study is VSC, allowing and independent control of active and reactive power. The active power exchange on the AC and DC side of the converter differ on losses. So, the AC power can be defined as a function of the DC power and converter losses, modelled according to a second order polynomial, as in [13]:

$$P_{lossvsc_i} = a + bI_{vsc_i} + cI^2_{vsc_i} \tag{18}$$

where a, b and c are p.u. parameters given by Table 1 and I_{vsc_i} represents the p.u. current flowing through the converter i:

$$I^*_{vsc_i} = \frac{\sqrt{P^2_{vsc_i} + Q^2_{vsc_i}}}{V_i} \tag{19}$$

As reflected in Table 1, two operating modes are distinguished for VSCs: rectifier and inverter.

VSC	a	b	c
Rectifier	11.033×10^{-3}	3.464×10^{-3}	5.40×10^{-3}
Inverter	11.033×10^{-3}	3.464×10^{-3}	7.67×10^{-3}

Table 1: Converter loss parameters [13]

2.6 Objective functions

The objective function presented in (2) can be chosen among several functions which are of interest in terms of operation or planning of the system. Some of them, are listed below:

Minimum power losses

$$[MIN]z = \sum_{j=1}^{p} \left(P_{g_j} - P_{d_j}\right) \tag{20}$$

Minimum generation costs

$$[MIN]z = \sum_{j=1}^{ng} C_i(P_i) \tag{21}$$

where ng represents the number of generators
Maximum reactive power margin

$$[MAX]z = \sum_{j=1}^{ng} Q_j \tag{22}$$

where ng represents the number of generators
Bus voltages closest to a profile

$$[MIN]z = \sum_{j=1}^{p} (V_j - V_{set})^2 \tag{23}$$

Minimum deviation from another state

$$[MIN]z = \sum_{j=1}^{p} (x_j - x_{set})^2 \tag{24}$$

3 System under study

The system used for this study is a 3 DC bus and 5 AC bus network, from [12, 18] sketched in Fig. 3 and Fig. 4. The DC and AC systems are linked through three VSC converters. The power in the DC network is injected into the AC grid through the connected inverters, responsible for the DC grid voltage control and which provide reactive power support to the AC grid when needed. The AC grid has two generators, one in bus 1 and another in bus 2. Loads are connected to buses 2, 3, 4 and 5. The DC grid is connected in AC buses 2, 3, and 5. The user needs to specify the control variables of the system (for example the generators injections, voltages setpoints)
The active and reactive power demand (loads) is defined by vectors P_d and Q_d in MW, and MVAr, respectively.

$$P_d = \left[0, 20, 45, 40, 60\right] \tag{25}$$

$$Q_d = \left[0, 10, 15, 5, 10\right] \tag{26}$$

As restrictions, the DC power flows are limited to 10 MW and the power converters rating is 20 MVA.

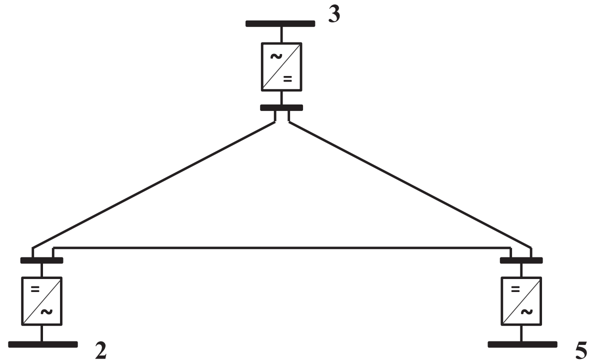

Fig. 3: 3 DC bus system

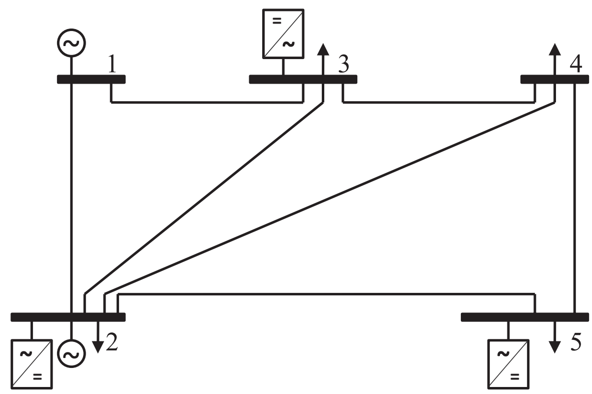

Fig. 4: 5 AC bus system

3.1 Minimum losses

This section shows the results of the optimization problem obtained when minimizing losses in the the hybrid DC/AC system sketched in Fig. 3 and. 4. The minimum losses of the whole system, computed as the difference between total active generation and total demand Equation (20)), are 5.52 MW. The active and reactive power flows are shown in Fig. 5 and 6. The DC and AC voltages on the different buses are reflected in Table 2.

Bus	AC voltage magnitude (p.u)	AC voltage angle(rad)	DC voltage of VSC
1	1.1000	0	-
2	1.1000	0	1.003
3	1.0809	-0.0430	1.000
4	1.0807	-0.0458	-
5	1.0811	-0.0468	0.999

Table 2: Bus voltages for loss minimisation

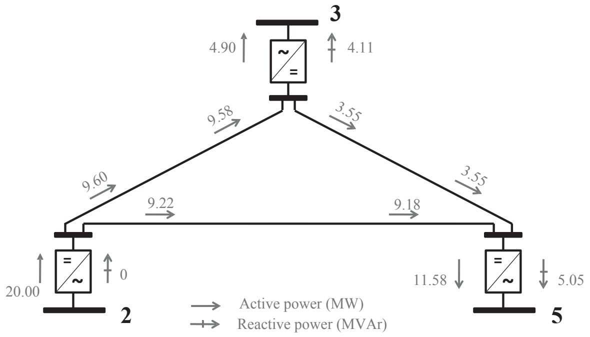

Fig. 5: Power flows in the 3 DC bus system for loss minimisation

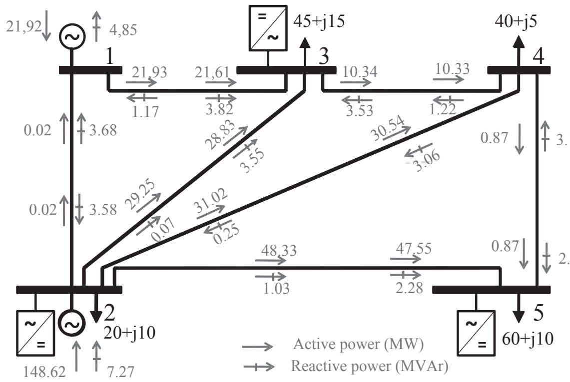

Fig. 6: Power flows in the 5 AC bus system for loss minimisation

3.2 Minimum deviation from a voltage profile

This section shows the results of the optimization problem obtained when minimizing the deviation of AC voltages from a preset voltage profile in the hybrid DC/AC system sketched in Fig. 3 and 4. The objective function (see (24)) is set so as to ensure that the system reaches the AC voltages specified by vector V_{set}, where

$$V_{set} = \left[1.08, 1.08, 1.05, 1.05, 1.05\right] \tag{27}$$

The objective function value resulting is 5.82×10^{-7}. The active and reactive power flows are shown in Fig. 7 and 8. The DC and AC voltages on the different buses are reflected in Table 3.

Bus	AC voltage magnitude (p.u)	AC voltage angle(rad)	DC voltage of VSC
1	1.0800	0	-
2	1.0799	-0.0223	0.9989
3	1.0496	-0.0663	1.0000
4	1.0506	-0.0709	-
5	1.0499	-0.0866	1.0006

Table 3: Bus voltages for minimum deviation from a voltage profile

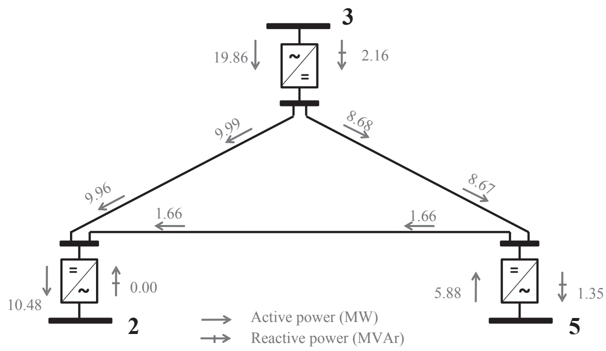

Fig. 7: Power flows in the 3 DC bus system for minimum deviation from a preset voltage

4 Sensitivity analysis

In this section, the effect of cable parameters and converter efficiency on loss minimization is analysed. For this purpose, the cable resistances (both DC and AC), as well as converter loss parameters values are varied +/- 10%. As shown in Figure 9, each parameter being analysed is multiplied by a correction factor to change its value between 90 and 110 %. The parameter values are represented on the x-axis and the corresponding objective function result is represented on the y-axis. It is worth mentioning that the effect of the parameters is very dependent on the system configuration.

In this case, the parameter showing the largest effect on total system losses is the AC cables resistance. The lower is the AC cable resistance, less power is flowing into the DC grid and more power is pushed to the AC grid. The DC resistance variation on +/- 10%, which has practically no effect on the objective function, does not change neither the power flows on the system.

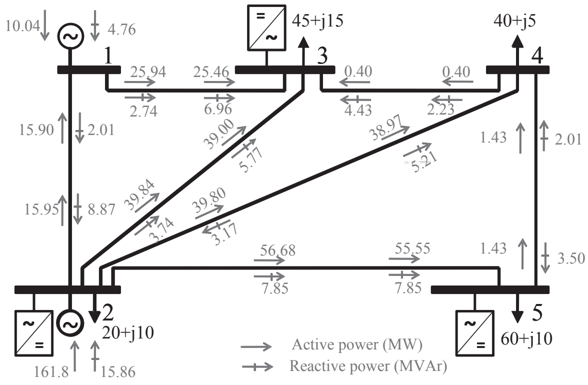

Fig. 8: Power flows in the 5 AC bus system for for minimum deviation from a preset voltage

When VSC losses increase, the power is injected from the DC grid to the AC grid decreases. As expected, for any change on the grid parameters, the power flows are adapted so that current follows the path that leads to the lowest losses.

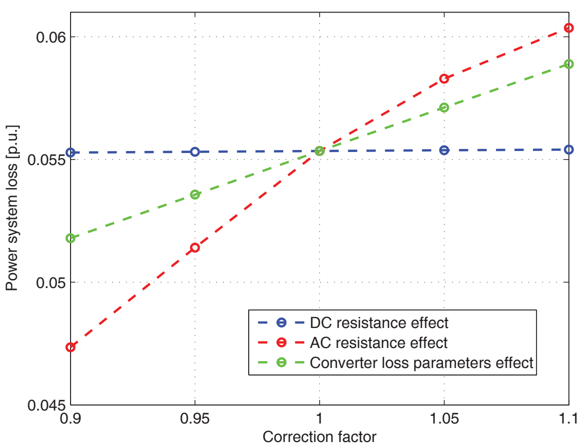

Fig. 9: Effect of DC resistance, AC resistance and converter loss parameters in power system loss

5 Conclusions

This paper presents an optimal power flow strategy to operate hybrid DC/AC systems. This methodology has been benchmarked with [18] in a particular study case consisting in a 5 AC bus connected to a 3 DC bus system, for minimum loss and minimum deviation from a preset voltage profile, leading to the same results. So, the tool proposed has been validated, showing a flexible methodology for analysing hybrid AC/DC grids optimal operation for several objective functions. Finally, sensitivity analysis allows to assess the effect of grid parameters on the objective function, shown in and example of loss minimization.

Acknowledgements

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